Modelling Interaction Between Unconfined Groundwater and Surface Water Bodies in Aquifers Subjected to Periodic Forcing

Centre for Groundwater Studies

Anthony J. Smith
(School of Biological and Environmental Sciences, Murdoch University, Western Australia, Fax: +61-9-310-4997, E-mail: asmith@essun1.murdoch.edu.au)

Lloyd R. Townley
(Centre for Groundwater Studies, CSIRO Division of Water Resources, Western Australia, Fax: +61-9-387-8211, E-mail: Lloyd.Townley@per.dwr.csiro.au)

CSIRO AUSTRALIA

1. Introduction and Objective

Cyclic responses to periodic sinusoidal forcing are well documented in the study of groundwater systems, however there are few results in 2D or 3D.

The objective of this research is to understand the dynamics of groundwater flow in a 2D vertical section beneath a wide shallow lake in a regional aquifer.

2. Background

Nield et al. [Water Resour. Res., 30(8), 1994] identified 39 fundamentally different groundwater flow patterns as a function of water body geometry and the relative magnitude of aquifer flows and net recharge to the water table. Steady flow patterns were classified as being recharge, discharge or flow-through.

Townley [Adv. Water Res., 18(3), 1995] presented analytical solutions for the response of 1D aquifers to periodic forcing.

Two further papers by Townley [in press] present analytical solutions for a 1D aquifer-lake-aquifer system, and a periodic finite element model called AQUIFEM-P for computing periodic fluctuations in heads and velocities in a 2D region.

This poster introduces ongoing research which uses AQUIFEM-P, with special mixed boundary conditions developed from Townley [1995], to simulate 2D flow patterns beneath a lake in a regional setting. It generalises the 1D aquifer-lake-aquifer results to 2D in the near field of a surface water body, and allows us to determine the extent to which the steady results obtained by Nield et al. [1994] can be applied to dynamic systems.

3. Conceptual Model and Solution Technique

Consider a cross-section through an aquifer, with groundwater flowing from a groundwater divide towards an ocean boundary (Figure 1a).

Because flow in the far field is essentially horizontal, we assume that these regions can be represented by 1D models with a transmissivity T which is spatially varying but constant in time (Figure 1b).

The near field is approximated as a rectangular region, following Nield et al. [1994]. The fact that the upper surface is horizontal and fixed in time is a geometric approximation which still allows many features of the true solution to be studied (Figure 1b).

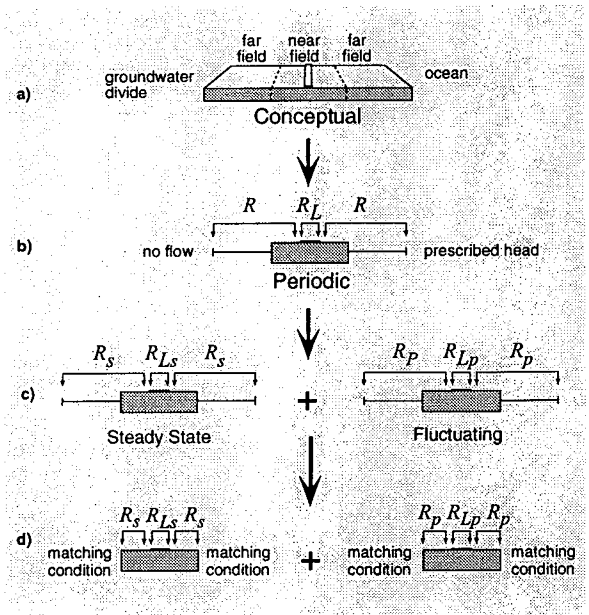


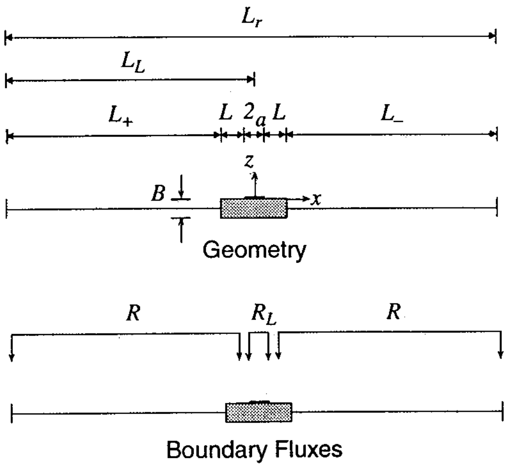
Figure 1 Problem Decomposition

Because the equations to be solved in the 1D and 2D regions are linear, we separate the response of the system into two parts: a steady solution, and a fluctuating (purely sinusoidal) solution (Figure 1c).

We use the analytical solutions of Townley [1995] to develop 3rd Type (mixed) boundary conditions at the ends of the 2D near field, such that these boundary or matching conditions encapsulate the steady and fluctuating behaviour of the far field (Figure 1d).

We solve both steady and periodic problems in 2D using AQUIFEM–P, and then use visualisation techniques to show pathlines and streaklines beneath the surface water body.

All results are presented in terms of key non-dimensional ratios (Figure 2). Aquifer properties include K_x, K_z, S_y and S_0, and the period of fluctuations is P.



Dimensional analysis

$$\text{geometry} = f\left\{\frac{2a}{B}, \frac{L}{B}, \frac{L_L}{L_r}, \frac{2_a}{L_L}\right\}$$

$$\text{steady solution} = f\left\{\frac{K_x}{K_z}, \frac{R_s L_r^2}{K_x B^2}, \frac{R_{Ls}}{Rs}\right\}$$

$$\text{transient solution} = f\left\{S_o B, S_y, \frac{L_r^2 S_y}{K_x BP}, \frac{|R_p| P}{S_y B}, \frac{|R_{Lp}| S_y}{|R_p|}, \arg\left(\frac{R_{Lp}}{R_p}\right), \arg(R_p)\right\}$$

Figure 2 Dimensional Analysis

4. Results

Because heads have steady and sinusoidal components, specific discharge and velocity also have both steady and sinusoidal components, in both x and z directions.

Sinusoidal variations in velocities result in velocity and displacement ellipses. When superimposed on steady flow, these result in pathlines of three kinds: smooth wavy pathlines, cuspy pathlines (with particles passing through instantaneous stagnation points) and loopy pathlines (Figure 3).

We have been keen to find out under what circumstances we would find complex cuspy or loopy flowlines in the real world. Systematic searching in the non-dimensional parameter space has started to provide some indications.

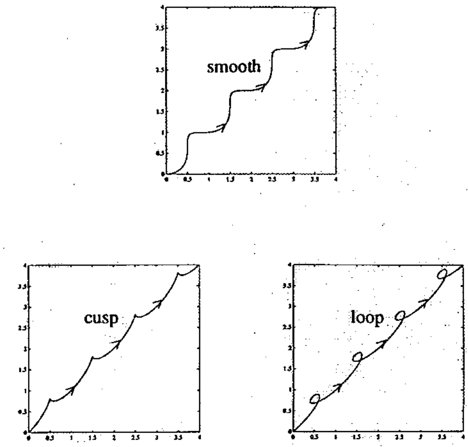


Figure 3 Types of Pathlines in Periodic Flow

Figures 4 and 5 show ellipses for lake-aquifer systems which are aquifer-driven and lake-driven, respectively. In Figure 4, the dominant fluctuation is backwards and forwards in the aquifer, with a flow-through fluctuation through the lake. In Figure 5, the dominant fluctuation is into and out of the aquifer, with the lake alternating between recharge and discharge behaviour. In both cases, ellipses are open only for intermediate values of $L_r^2 S_y / K_x BP$ (i.e. for aquifer response times which are of the same order as the period of fluctuations), and for large values of $|R_p| P / S_y B$, i.e. when the fluctuation in water table due to recharge fluctuations is significant relative to aquifer thickness. While these results may be intuitively reasonable, they have not previously been described in quantitative terms.

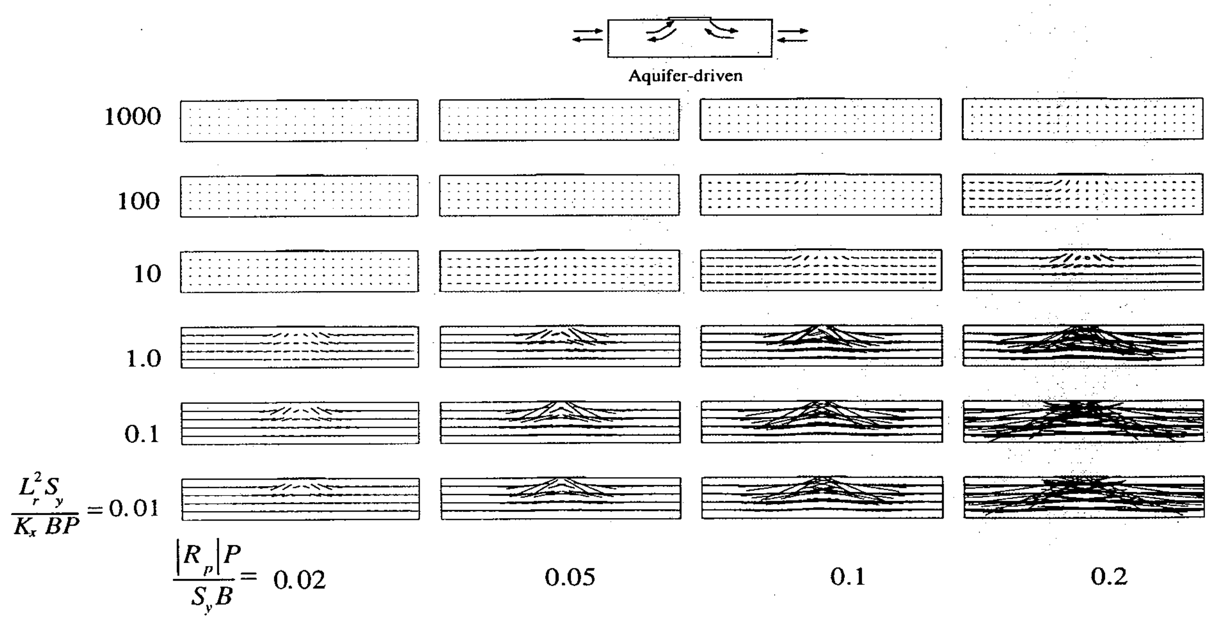


Figure 4 Fluctuating Response in Aquifer-driven System

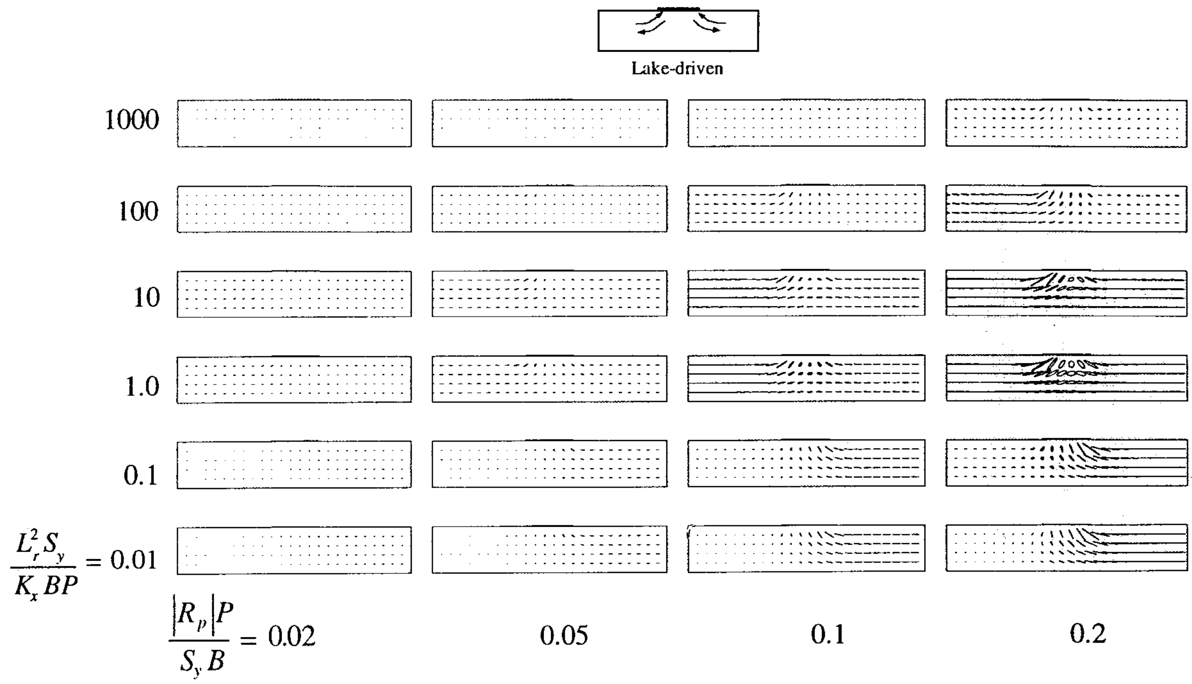


Figure 5 Fluctuating Response in Lake-driven System

Consider a steady flow pattern from left to right which in Nield et al.'s [1994] classification would be described as a flow-through regime of type FT2 (Figure 6a). With strong sinusoidal flow into and out of the lake, AQUIFEM-P predicts velocity and/or displacement ellipses as shown in Figure 6b. Superimposing the steady and fluctuating velocity fields and starting particles at a number of times through a period shows that particles can be completely captured by the lake (Figure 6c), or they can migrate from the lake into the lake's release zone (Figure 6d), or they can appear to be released by the lake, only to be recaptured at a later time (Figure 6e).

5. Discussion and Conclusions

Groundwater flow patterns beneath surface water bodies during periods of dynamic changes in water levels may be much more complex than previously realised.

The complex advective flow patterns beneath surface water bodies may be responsible for effects that would commonly be described as mixing or dispersion.

A sinusoidal dynamic is one of the simplest to deal with mathematically, thus AQUIFEM-P provides a versatile tool for studying the details of flow patterns in vertical section.

Special mixed boundary conditions at the ends of the near field, allow AQUIFEM-P to account for regional behaviour.

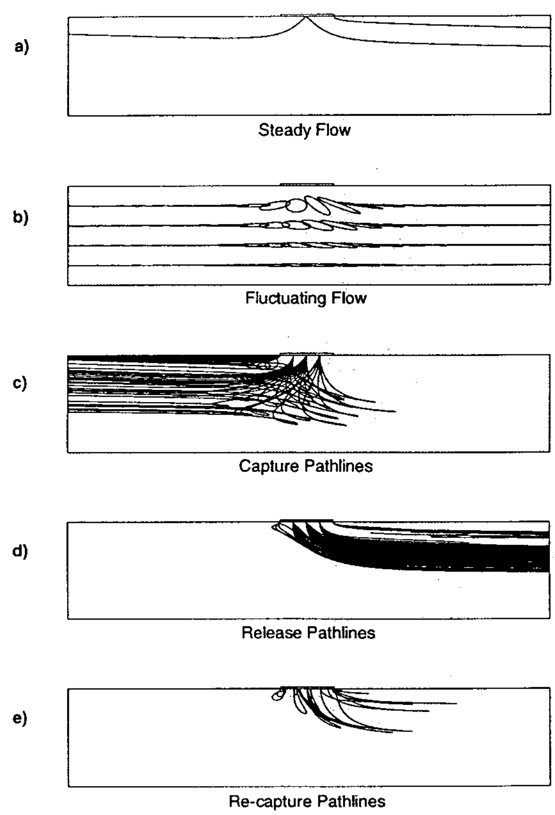


Figure 6 Example Pathlines with Strong Lake forcing